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MA222 - Computational Linear Algebra
Problem Sheet - 2

Exploiting Structure

1. Give an algorithm that overwrites A with A^2 where $A \in \mathbb{R}^{n\times n}$ is (a) upper triangular and (b) square. Strive for a minimum workspace in each case.

2. Suppose $A \in \mathbb{R}^{n\times n}$ is upper Hessenberg and that scalars $\lambda_1, \ldots, \lambda_T$ are given. Give a saxpy algorithm for computing the first column of $M = (A - \lambda_1 I) \cdots (A - \lambda_T I)$.

3. Give a column saxpy algorithm for the n-by-n matrix multiplication problem $C = AB$ where A is upper triangular and B is lower triangular.

4. Extend Algorithm 1.2.2 so that it can handle rectangular band matrices. Be sure to describe the underlying data structure.

5. $A \in \mathbb{R}^{n\times n}$ is Hermitian if $A^H = A$. If $A = B + iC$, then it is easy to show that $B^T = B$ and $C^T = -C$. Suppose we represent A in an array $A.herm$ with the property that $A.herm(i, j)$ however b_{ij} if $i \geq j$ and c_{ij} if $j > i$. Using this data structure write a matrix-vector multiply function that computes $Re(z)$ and $Im(z)$ from $Re(x)$ and $Im(x)$ so that $z = Ax$.

6. Suppose $X \in \mathbb{R}^{n\times p}$ and $A \in \mathbb{R}^{n\times n}$, with A symmetric and stored by diagonal. Give an algorithm that computes $Y = X^T AX$ and stores the result by diagonal. Use separate arrays for A and Y.

7. Suppose $a \in \mathbb{R}^n$ is given and that $A \in \mathbb{R}^{n\times n}$ has the property that $a_{ij} = a_{|i-j|+1}$. Give an algorithm that overwrites y with $Ax + y$ where $x, y \in \mathbb{R}^n$ are given.

8. Suppose $a \in \mathbb{R}^n$ is given and that $A \in \mathbb{R}^{n\times n}$ has the property that $a_{ij} = a_{((i+j-1) \mod n)+1}$. Give an algorithm that overwrites y with $Ax + y$ where $x, y \in \mathbb{R}^n$ are given.

9. Develop a compact store-by-diagonal scheme for unsymmetric band matrices and write the corresponding gaxpy algorithm.

10. Suppose p and q are n-vectors and that $A = (a_{ij})$ is defined by $a_{ij} = a_{ji} = p_i q_j$ for $1 \leq i \leq j \leq n$. How many flops are required to compute $y = Ax$ where $x \in \mathbb{R}^n$ is given?

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